

Massive Text Indices

Florian Kurpicz

Departments of Computer Science
KIT & TU Dortmund

Algorithms for Big Data
DFG Schwerpunktprogramm 1736

Frankfurt, September 29th – October 01st, 2014

Texts are Important

Introduction

A Showcase for Big Data

- ▶ World Wide Web
- ▶ digital libraries
- ▶ DNA and proteins

Texts are Important

Introduction

A Showcase for Big Data

- ▶ World Wide Web
- ▶ digital libraries
- ▶ DNA and proteins

What to do with the Text

- ▶ Store
- ▶ Structure
- ▶ Search
- ▶ Compress

And all that efficiently.

Texts are Important

Introduction

A Showcase for Big Data

- ▶ World Wide Web
- ▶ digital libraries
- ▶ DNA and proteins

What to do with the Text

- ▶ Store
- ▶ Structure
- ▶ Search
- ▶ Compress

And all that **efficiently**.

Texts are Important

Introduction

A Showcase for Big Data

- ▶ World Wide Web
- ▶ digital libraries
- ▶ DNA and proteins

What to do with the Text

- ▶ Store
- ▶ Structure
- ▶ Search
- ▶ Compress

And all that efficiently.

Texts are Important

Introduction

A Showcase for Big Data

- ▶ World Wide Web
- ▶ digital libraries
- ▶ DNA and proteins

What to do with the Text

- ▶ Store
- ▶ Structure
- ▶ Search
- ▶ Compress

And all that efficiently.

The Amount of Data

- ▶ Increases faster than the storage and computation capacities.
- ▶ 10 years ago: The human genome (~ 5.8 Gigabits ≈ 725 Megabyte).
- ▶ Today: Analyse 1000 human genomes simultaneously.

Texts are Important

Introduction

A Showcase for Big Data

- ▶ World Wide Web
- ▶ digital libraries
- ▶ DNA and proteins

What to do with the Text

- ▶ Store
- ▶ Structure
- ▶ Search
- ▶ Compress

And all that efficiently.

The Amount of Data

- ▶ Increases faster than the storage and computation capacities.
- ▶ 10 years ago: The human genome (~ 5.8 Gigabits ≈ 725 Megabyte).
- ▶ Today: Analyse 1000 human genomes simultaneously.
- ▶ Tomorrow: Analyse 1,000,000 human genomes simultaneously?

What do we want?

The Problem

Given a text T and a pattern P find all occurrences of P in T .

T

What do we want?

The Problem

Given a text T and a pattern P find all occurrences of P in T .

T



What do we want?

The Problem

Given a text T and a pattern P find all occurrences of P in T .

T

What do we want?

The Problem

Given a text T and a pattern P find all occurrences of P in T .

T

Terminology

- ▶ Let T be a string over an alphabet Σ of length n .
- ▶ $S_i = T[i, n]$ is a suffix of T starting at index i .
- ▶ $P_i = T[1, i]$ is a prefix of T of length i .

What do we want?

The Problem

Given a text T and a pattern P find all occurrences of P in T .

T

Terminology

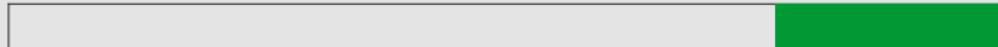
- ▶ Let T be a string over an alphabet Σ of length n .
- ▶ $S_i = T[i, n]$ is a **suffix** of T starting at index i .
- ▶ $P_i = T[1, i]$ is a prefix of T of length i .

What do we want?

The Problem

Given a text T and a pattern P find all occurrences of P in T .

T



Terminology

- ▶ Let T be a string over an alphabet Σ of length n .
- ▶ $S_i = T[i, n]$ is a **suffix** of T starting at index i .
- ▶ $P_i = T[1, i]$ is a prefix of T of length i .

What do we want?

The Problem

Given a text T and a pattern P find all occurrences of P in T .

T

Terminology

- ▶ Let T be a string over an alphabet Σ of length n .
- ▶ $S_i = T[i, n]$ is a suffix of T starting at index i .
- ▶ $P_i = T[1, i]$ is a prefix of T of length i .

What do we want?

The Problem

Given a text T and a pattern P find all occurrences of P in T .

T

Terminology

- ▶ Let T be a string over an alphabet Σ of length n .
- ▶ $S_i = T[i, n]$ is a suffix of T starting at index i .
- ▶ $P_i = T[1, i]$ is a **prefix** of T of length i .

What do we want?

The Problem

Given a text T and a pattern P find all occurrences of P in T .

T



Terminology

- ▶ Let T be a string over an alphabet Σ of length n .
- ▶ $S_i = T[i, n]$ is a suffix of T starting at index i .
- ▶ $P_i = T[1, i]$ is a **prefix** of T of length i .

What do we want?

The Problem

Given a text T and a pattern P find all occurrences of P in T .

T

Terminology

- ▶ Let T be a string over an alphabet Σ of length n .
- ▶ $S_i = T[i, n]$ is a suffix of T starting at index i .
- ▶ $P_i = T[1, i]$ is a prefix of T of length i .

What do we want?

The Problem

Given a text T and a pattern P find all occurrences of P in T .

T

Terminology

- ▶ Let T be a string over an alphabet Σ of length n .
- ▶ $S_i = T[i, n]$ is a suffix of T starting at index i .
- ▶ $P_i = T[1, i]$ is a prefix of T of length i .

How Fast?

- ▶ Without preprocessing: $\Theta(|T| + |P|)$ [Knuth, Morris, Pratt, 1977]
- ▶ With Preprocessing: $\Theta(|P|) + \text{Preprocessing}$ [Weiner, 1973]

What do we want?

The Problem

Given a text T and a pattern P find all occurrences of P in T .

T

Terminology

- ▶ Let T be a string over an alphabet Σ of length n .
- ▶ $S_i = T[i, n]$ is a suffix of T starting at index i .
- ▶ $P_i = T[1, i]$ is a prefix of T of length i .

How Fast?

- ▶ Without preprocessing: $\Theta(|T| + |P|)$ [Knuth, Morris, Pratt, 1977]
- ▶ With Preprocessing: $\Theta(|P|) + \text{Preprocessing}$ [Weiner, 1973]

Build an Index

Indices and Texts

Structured Text vs. Unstructured Text

- ▶ Linguistic text
- ▶ Source Code
- ▶ DNA sequences
- ▶ Executables

Indices and Texts

Structured Text vs. Unstructured Text

- ▶ Linguistic text
- ▶ Source Code
- ▶ DNA sequences
- ▶ Executables

Word-Based vs. Full-Text Indices

	Unstructured	Structured
Word-Based		
Full-Text		

Indices and Texts

Structured Text vs. Unstructured Text

- ▶ Linguistic text
- ▶ Source Code
- ▶ DNA sequences
- ▶ Executables

Word-Based vs. Full-Text Indices

	Unstructured	Structured
Word-Based		
Full-Text		

Indices and Texts

Structured Text vs. Unstructured Text

- ▶ Linguistic text
- ▶ Source Code
- ▶ DNA sequences
- ▶ Executables

Word-Based vs. Full-Text Indices

	Unstructured	Structured
Word-Based		
Full-Text		

A Word-Based Index

Index of “Algorithms on String” (Crochemore, 2007)

S suffix, 3

suffix array, 146, 158-174, 220, 222, 230, 354

SUFFIX-AUTO, 205

suffix tree, 184-193, 221, 223, 303, 305

T trie, 56

TRIE, 56

Tromp, 328

turbo-shift, 112

A Word-Based Index

Index of “Algorithms on String” (Crochemore, 2007)

S suffix, 3

suffix array, 146, 158-174, 220, 222, 230, 354

SUFFIX-AUTO, 205

suffix tree, 184-193, 221, 223, 303, 305

T trie, 56

TRIE, 56

Tromp, 328

turbo-shift, 112

No Match = No Reference

- We want every occurrence of P = tree.

A Word-Based Index

Index of “Algorithms on String” (Crochemore, 2007)

S suffix, 3

suffix array, 146, 158-174, 220, 222, 230, 354

SUFFIX-AUTO, 205

suffix tree, 184-193, 221, 223, 303, 305

T trie, 56

TRIE, 56

Tromp, 328

turbo-shift, 112

No Match = No Reference

- We want every occurrence of P = tree.

A Word-Based Index

Index of “Algorithms on String” (Crochemore, 2007)

S suffix, 3

suffix array, 146, 158-174, 220, 222, 230, 354

SUFFIX-AUTO, 205

suffix **tree**, 184-193, 221, 223, 303, 305

T trie, 56

TRIE, 56

Tromp, 328

turbo-shift, 112

No Match = No Reference

- We want every occurrence of $P = \text{tree}$.

A Word-Based Index

Index of “Algorithms on String” (Crochemore, 2007)

S suffix, 3

suffix array, 146, 158-174, 220, 222, 230, 354

SUFFIX-AUTO, 205

suffix tree, 184-193, 221, 223, 303, 305

T trie, 56

TRIE, 56

Tromp, 328

turbo-shift, 112

No Match = No Reference

- We want every occurrence of P = tree.
- **Build an index containing all words?**

A Word-Based Index

Index of “Algorithms on String” (Crochemore, 2007)

S suffix, 3

suffix array, 146, 158-174, 220, 222, 230, 354

SUFFIX-AUTO, 205

suffix tree, 184-193, 221, 223, 303, 305

T trie, 56

TRIE, 56

Tromp, 328

turbo-shift, 112

No Match = No Reference

- We want every occurrence of P = tree.
- **Build an index containing all words?**

A Word-Based Index

Index of “Algorithms on String” (Crochemore, 2007)

S suffix, 3

suffix array, 146, 158-174, 220, 222, 230, 354

SUFFIX-AUTO, 205

suffix tree, 184-193, 221, 223, 303, 305

T trie, 56

TRIE, 56

Tromp, 328

turbo-shift, 112

No Match = No Reference

- ▶ We want every occurrence of $P = \text{tree}$.
- ▶ **Build an index containing all words suffixes?**

Unstructured Text

DNA as Text

```
A C G T T G T T C T G A A G A A A A T T T A T G  
A A C G T T A T A C A A C G G T C G A C C A T A  
G G A T T A C A C G G C A G A G G T G G T T G T  
C T A A G G C G T T A C C C C A A T C G T T A T  
A A C G C T A C A T G C A C G A A A C C A A G T  
G G A C A T A G C C T T G T A G C G T T A A T G  
G A G C C T T C A C C G G C A T T C T G T T T A  
A T C A T A T G C A G A T C C G T T A T G C G C  
C T C T G T A A T A G G G A C T A A A A A A G T
```

Unstructured Text

DNA as Text

```
A C G T T G T T C T G A A G A A A A T T T A T G  
A A C G T T A T A C A A C G G T C G A C C A T A  
G G A T T A C A C G G C A G A G G T G G T T G T  
C T A A G G C G T T A C C C C A A T C G T T A T  
A A C G C T A C A T G C A C G A A A C C A A G T  
G G A C A T A G C C T T G T A G C G T T A A T G  
G A G C C T T C A C C G G C A T T C T G T T T A  
A T C A T A T G C A G A T C C G T T A T G C G C  
C T C T G T A A T A G G G A C T A A A A A A G T
```

$$P_0 = \text{CGTTA}$$

Unstructured Text

DNA as Text

A C G T T G T T C T G A A G A A A A T T T A T G
A A C **C G T T A** T A C A A C G G T C G A C C A T A
G G A T T A C A C G G C A G A G G T G G T T G T
C T A A G G **C G T T A** C C C C A A T **C G T T A** T
A A C G C T A C A T G C A C G A A A C C A A G T
G G A C A T A G C C T T G T A G **C G T T A** A T G
G A G C C T T C A C C G G C A T T C T G T T T A
A T C A T A T G C A G A T C **C G T T A** T G C G C
C T C T G T A A T A G G G A C T A A A A A A G T

$$P_0 = \text{CGTTA}$$

Unstructured Text

DNA as Text

```
A C G T T G T T C T G A A G A A A A T T T A T G  
A A C C G T T A T A C A A C G G T C G A C C A T A  
G G A T T A C A C G G C A G A G G T G G T T G T  
C T A A G G C G T T A C C C C A A T C G T T A T  
A A C G C T A C A T G C A C G A A A C C A A G T  
G G A C A T A G C C T T G T A G C G T T A A T G  
G A G C C T T C A C C G G C A T T C T G T T T A  
A T C A T A T G C A G A T C C G T T A T G C G C  
C T C T G T A A T A G G G A C T A A A A A A G T
```

$$\begin{aligned}P_0 &= \text{CGTTA} \\P_1 &= \dots\end{aligned}$$

Combining Index- and Text-Types

Word-Based vs. Full-Text Indices

	Unstructured	Structured
Word-Based	X	
Full-Text		

Combining Index- and Text-Types

Word-Based vs. Full-Text Indices

	Unstructured	Structured
Word-Based	✗	✓
Full-Text		

Combining Index- and Text-Types

Word-Based vs. Full-Text Indices

	Unstructured	Structured
Word-Based	X	✓
Full-Text	✓	✓

Combining Index- and Text-Types

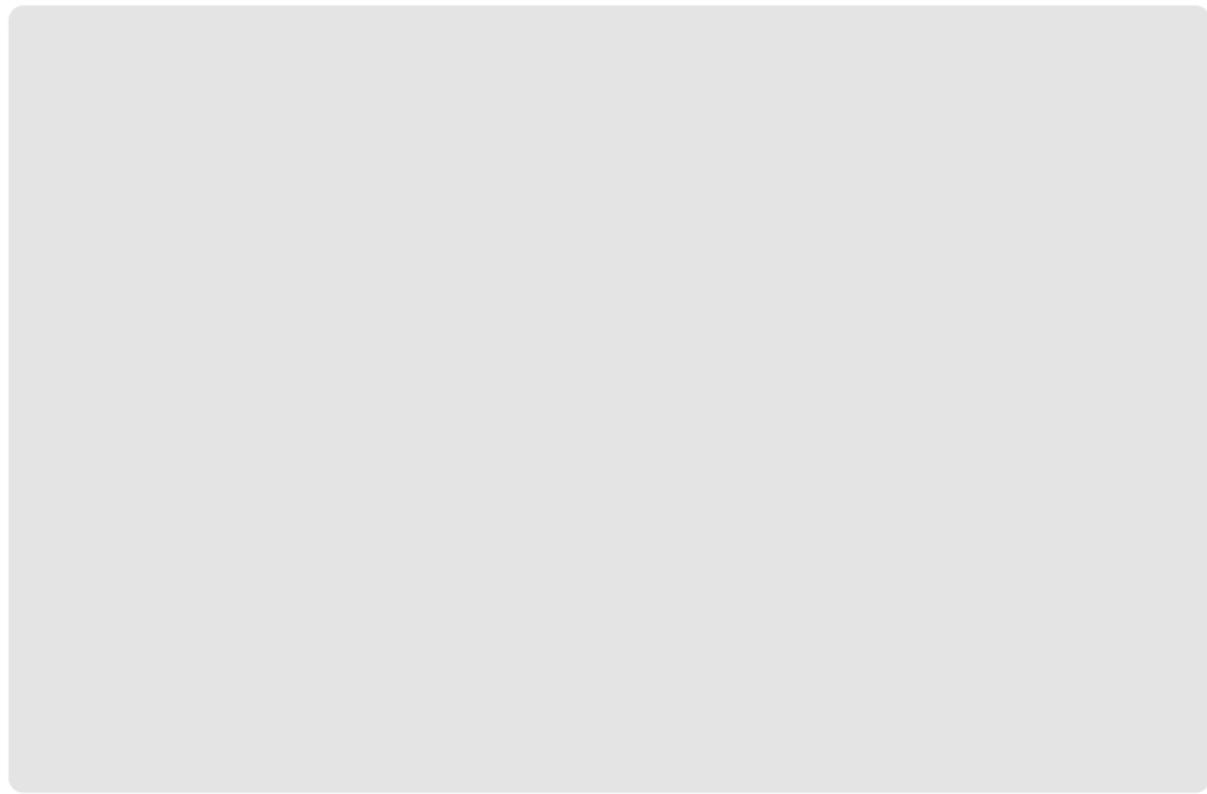
Word-Based vs. Full-Text Indices

	Unstructured	Structured
Word-Based	X	✓
Full-Text	✓	✓

We want full-text indices!

How to Represent a Fulltext Index

Utilising Indices



How to Represent a Fulltext Index

- A AAAAAAGT, 208
AAAAAGT, 209
AAAAGT, 210
...
C CAACGGTCGACCATAGGATTAC[...]AAAAAAAGT, 33
CAAGTGGACATAGCCTTAGC[...]AAAAAAAGT, 115
CAATCGTTATAACGCTACATGC[...]AAAAAAAGT, 86
...
G GAAAATTTATGAACGTTATACA[...]AAAAAAAGT, 13
GAAACCAAGTGGACATAGCCTT[...]AAAAAAAGT, 110
GAACGTTATACAACGGTCGACC[...]AAAAAAAGT, 23
...
T T, 215
TAAAAAAAGT, 207
TAACGCTACATGCACGAAACCA[...]AAAAAAAGT, 95
...

How to Represent a Fulltext Index

A AAAAAAGT, 208
AAAAAGT, 209
AAAAGT, 210
...

Space $O(|T|^2)$

C CAACGGTCGACCATAGGATTAC[...]AAAAAAAGT, 33
CAAGTGGACATAGCCTTAGC[...]AAAAAAAGT, 115
CAATCGTTATAACGCTACATGC[...]AAAAAAAGT, 86
...

G GAAAATTTATGAACGTTATACA[...]AAAAAAAGT, 13
GAAACCAAGTGGACATAGCCTT[...]AAAAAAAGT, 110
GAACGTTATACAACGGTCGACC[...]AAAAAAAGT, 23
...

T T, 215
TAAAAAAAGT, 207
TAACGCTACATGCACGAAACCA[...]AAAAAAAGT, 95
...

How to Represent a Fulltext Index

A AAAAAAGT, 208
AAAAAGT, 209
AAAAGT, 210

Space $O(|T|^2)$

C CAACTGTCGACCATAGGATTAC[...]AAAAAGT, 33
CAAGTG[...]CATAGCCTTGTAC[...]AAAAAGT, 115
CAATCGTTA[...]ACGCTACG[...]AAAAAGT, 86
...

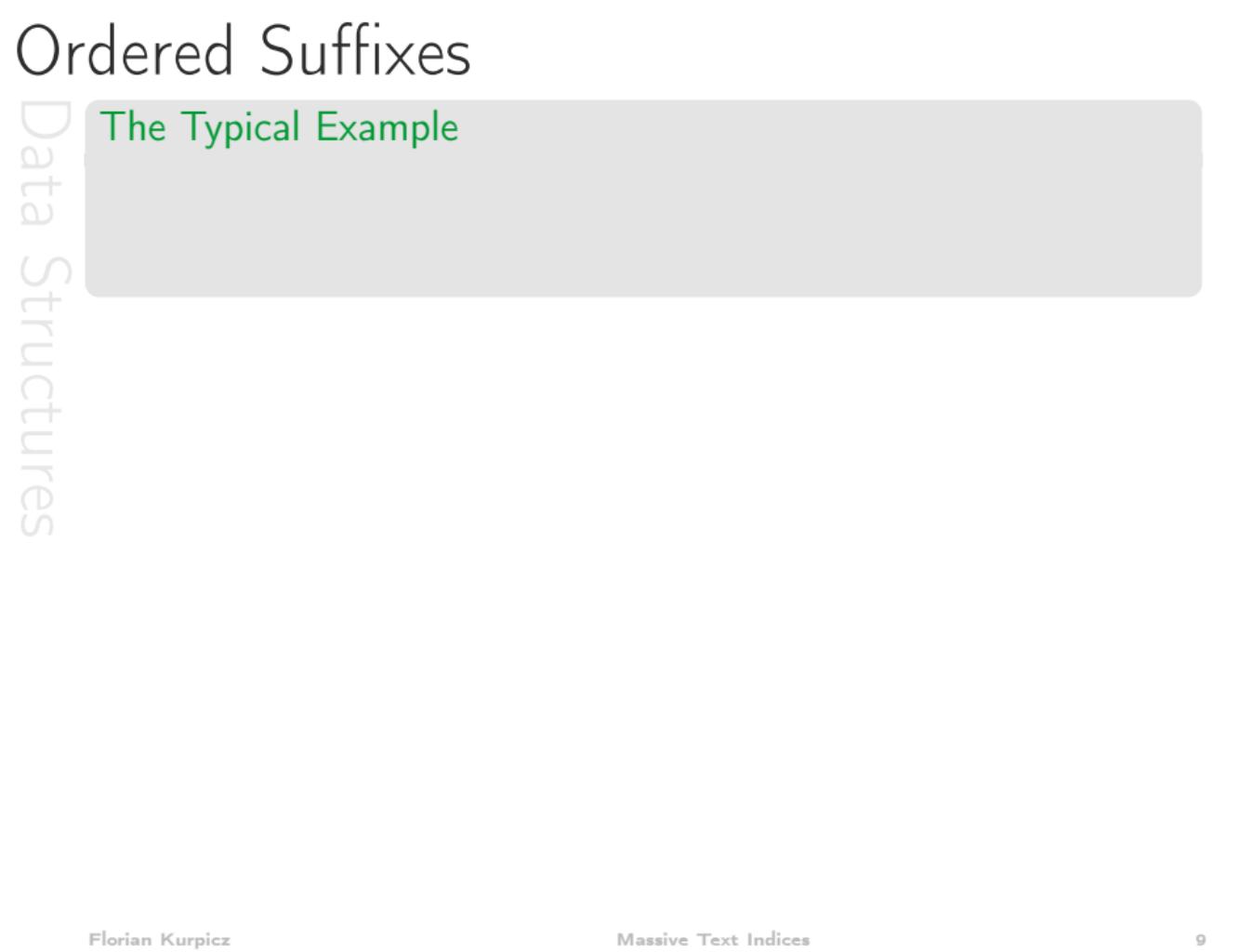
G GAAAATTTATCGCTGTTACA[...]AAAAAGT, 13
GAAACCAA[...]GGACATAGCT[...]AAAAAGT, 110
GAACCTATACAACGGTCGACC[...]AAAAAGT, 23
...

I, 215

TAAAAAAAGT, 207

TAACGCTACATGCACGAAACCA[...]AAAAAGT, 95

...



Ordered Suffixes

The Typical Example

Data Structures

Ordered Suffixes

The Typical Example



Ordered Suffixes

The Typical Example


$$T = \begin{smallmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{B} & \text{A} & \text{N} & \text{A} & \text{N} & \text{A} & \$ \end{smallmatrix}$$

Ordered Suffixes

The Typical Example


$$T = \overset{0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6}{\text{BANANA\$}}$$

Suffixarray (SA)

i	0	1	2	3	4	5	6
	\$	A	A	A	B	N	N
	\$	N	N	A	A	A	
	A	A	N		\$	N	
	\$	N	A			A	
	A	N					\$
	\$	A					

- ▶ Array containing the suffixes of a string T in lexicographical order.

Ordered Suffixes

The Typical Example


$$T = \overset{0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6}{\text{BANANA\$}}$$

Suffixarray (SA)

i	0	1	2	3	4	5	6
	\$	A	A	A	B	N	N
	\$	N	N	A	A	A	
	A	A	N		\$	N	
	\$	N	A			A	
	A	N				\$	
	\$	A					
		\$					

- ▶ Array containing the suffixes of a string T in lexicographical order.
- ▶ Suffixes are represented by their index.

Ordered Suffixes

The Typical Example


$$T = \overset{0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6}{\text{BANANA\$}}$$

Suffixarray (SA)

i	0	1	2	3	4	5	6
SA[i]	6	5	3	1	0	4	2
\$	A	A	A	B	N	N	
\$	N	N	A	A	A	A	
	A	A	N	\$	N		
\$	N	A				A	
	A	N					\$
\$	A						
	\$						

- ▶ Array containing the suffixes of a string T in lexicographical order.
- ▶ Suffixes are represented by their index.

Ordered Suffixes

The Typical Example


$$T = \overset{0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6}{\text{BANANA\$}}$$

Suffixarray (SA)

i	0	1	2	3	4	5	6
SA[i]	6	5	3	1	0	4	2
\$	A	A	A	B	N	N	
\$	N	N	A	A	A		
	A	A	N		\$	N	
\$	N	A				A	
	A	N					\$
\$	A						
	\$						

- ▶ Array containing the suffixes of a string T in lexicographical order.
- ▶ Suffixes are represented by their index.
- ▶ Answer queries with binary search.

Ordered Suffixes

The Typical Example


$$T = \overset{0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6}{\text{BANANA\$}}$$

Suffixarray (SA)

i	0	1	2	3	4	5	6
SA[i]	6	5	3	1	0	4	2
\$	A	A	A	B	N	N	
\$	N	N	A	A	A	A	
	A	A	N	\$	N		
\$	N	A			A		
	A	N				\$	
\$	A						
	\$						

- ▶ Array containing the suffixes of a string T in lexicographical order.
- ▶ Suffixes are represented by their index.
- ▶ Answer queries with binary search.
- ▶ $P = \text{BAN}$

Ordered Suffixes

The Typical Example


$$T = \overset{0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6}{\text{BANANA\$}}$$

Suffixarray (SA)

i	0	1	2	3	4	5	6
SA[i]	6	5	3	1	0	4	2
\$	A	A	A	B	N	N	
\$	N	N	A	A	A	A	
	A	A	N	\$	N		
\$	N	A			A		
	A	N				\$	
\$	A						
	\$						



- ▶ Array containing the suffixes of a string T in lexicographical order.
- ▶ Suffixes are represented by their index.
- ▶ Answer queries with binary search.
- ▶ $P = \text{BAN}$

Ordered Suffixes

The Typical Example


$$T = \overset{0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6}{\text{BANANA\$}}$$

Suffixarray (SA)

i	0	1	2	3	4	5	6
SA[i]	6	5	3	1	0	4	2
\$	A	A	A	B	N	N	
\$	N	N	A	A	A		
	A	A	N		\$	N	
\$	N	A				A	
	A	N					\$
\$	A						
	\$						

- ▶ Array containing the suffixes of a string T in lexicographical order.
- ▶ Suffixes are represented by their index.
- ▶ Answer queries with binary search.
- ▶ $P = \text{BAN}$

Ordered Suffixes

The Typical Example


$$T = \begin{smallmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{B} & \text{A} & \text{N} & \text{A} & \text{N} & \text{A} & \$ \end{smallmatrix}$$

Suffixarray (SA)

i	0	1	2	3	4	5	6
SA[i]	6	5	3	1	0	4	2
\$	A	A	A	B	N	N	
\$	N	N	A	A	A		
	A	A	N	\$	N		
\$	N	A			A		
	A	N				\$	
\$	A						
	\$						

- ▶ Array containing the suffixes of a string T in lexicographical order.
- ▶ Suffixes are represented by their index.
- ▶ Answer queries with binary search.
- ▶ $P = \text{BAN}$

Common Prefixes

The Typical Example


$$T = \begin{smallmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{B} & \text{A} & \text{N} & \text{A} & \text{N} & \text{A} & \$ \end{smallmatrix}$$

Common Prefixes

The Typical Example


$$T = \begin{smallmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{B} & \text{A} & \text{N} & \text{A} & \text{N} & \text{A} & \$ \end{smallmatrix}$$

Longest-Common-Prefix Array (LCP)

i	0	1	2	3	4	5	6
SA[i]	6	5	3	1	0	4	2
LCP[i]							
	\$	A	A	A	B	N	N
	\$	N	N	A	A	A	
	A	A	N		\$	N	
	\$	N	A			A	
		A	N				\$
	\$	A					
		\$					

Common Prefixes

The Typical Example


$$T = \begin{smallmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{BANANA\$} \end{smallmatrix}$$

Longest-Common-Prefix Array (LCP)

i	0	1	2	3	4	5	6
$\text{SA}[i]$	6	5	3	1	0	4	2
$\text{LCP}[i]$							
	\$	A	A	A	B	N	N
	\$	N	N	A	A	A	
	A	A	N		\$	N	
	\$	N	A			A	
	A	N				\$	
	\$	A					
		\$					

- ▶ Array containing the size of the longest common prefix of $S_{\text{SA}[i-1]}$ and $S_{\text{SA}[i]}$ at position i .

Common Prefixes

The Typical Example


$$T = \begin{smallmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{B} & \text{A} & \text{N} & \text{A} & \text{N} & \text{A} & \$ \end{smallmatrix}$$

Longest-Common-Prefix Array (LCP)

i	0	1	2	3	4	5	6
$\text{SA}[i]$	6	5	3	1	0	4	2
$\text{LCP}[i]$							
\$	A	A	A	B	N	N	
\$	N	N	A		A	A	
A	A	N			\$	N	
\$	N	A				A	
A	N						\$
\$	A						
	\$						

- ▶ Array containing the size of the longest common prefix of $S_{\text{SA}[i-1]}$ and $S_{\text{SA}[i]}$ at position i .

Common Prefixes

The Typical Example


$$T = \begin{smallmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{B} & \text{A} & \text{N} & \text{A} & \text{N} & \text{A} & \$ \end{smallmatrix}$$

Longest-Common-Prefix Array (LCP)

i	0	1	2	3	4	5	6
$\text{SA}[i]$	6	5	3	1	0	4	2
$\text{LCP}[i]$							2
	\$	A	A	A	B	N	N
	\$	N	N	A	A	A	
	A	A	N		\$	N	
	\$	N	A			A	
	A	N					\$
	\$	A					
	\$						

- ▶ Array containing the size of the longest common prefix of $S_{\text{SA}[i-1]}$ and $S_{\text{SA}[i]}$ at position i .

Common Prefixes

The Typical Example


$$T = \begin{smallmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{BANANA\$} \end{smallmatrix}$$

Longest-Common-Prefix Array (LCP)

i	0	1	2	3	4	5	6
$\text{SA}[i]$	6	5	3	1	0	4	2
$\text{LCP}[i]$							2
	\$	A	A	A	B	N	N
	\$	N	N	A	A	A	
	A	A	N		\$	N	
	\$	N	A			A	
		A	N				\$
	\$	A					
		\$					

- ▶ Array containing the size of the longest common prefix of $S_{\text{SA}[i-1]}$ and $S_{\text{SA}[i]}$ at position i .

Common Prefixes

The Typical Example


$$T = \begin{smallmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{B} & \text{A} & \text{N} & \text{A} & \text{N} & \text{A} & \$ \end{smallmatrix}$$

Longest-Common-Prefix Array (LCP)

i	0	1	2	3	4	5	6
$\text{SA}[i]$	6	5	3	1	0	4	2
$\text{LCP}[i]$							2
	\$	A	A	A	B	N	N
	\$	N	N	A	A	A	
	A	A	N		\$	N	
	\$	N	A			A	
		A	N				\$
		\$	A				
			\$				

- ▶ Array containing the size of the longest common prefix of $S_{\text{SA}[i-1]}$ and $S_{\text{SA}[i]}$ at position i .

Common Prefixes

The Typical Example


$$T = \begin{smallmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{B} & \text{A} & \text{N} & \text{A} & \text{N} & \text{A} & \$ \end{smallmatrix}$$

Longest-Common-Prefix Array (LCP)

i	0	1	2	3	4	5	6
$\text{SA}[i]$	6	5	3	1	0	4	2
$\text{LCP}[i]$						0	2
	\$	A	A	A	B	N	N
	\$	N	N	A	A	A	
	A	A	N		\$	N	
	\$	N	A			A	
		A	N				\$
		\$	A				
			\$				

- ▶ Array containing the size of the longest common prefix of $S_{\text{SA}[i-1]}$ and $S_{\text{SA}[i]}$ at position i .

Common Prefixes

The Typical Example


$$T = \overset{0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6}{\text{BANANA\$}}$$

Longest-Common-Prefix Array (LCP)

i	0	1	2	3	4	5	6
$\text{SA}[i]$	6	5	3	1	0	4	2
$\text{LCP}[i]$						0	2
	\$	A	A	A	B	N	N
	\$	N	N	A	A	A	
	A	A	N		\$	N	
	\$	N	A			A	
	A	N				\$	
	\$	A					
		\$					

- ▶ Array containing the size of the longest common prefix of $S_{\text{SA}[i-1]}$ and $S_{\text{SA}[i]}$ at position i .

Common Prefixes

The Typical Example


$$T = \overset{0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6}{\text{BANANA\$}}$$

Longest-Common-Prefix Array (LCP)

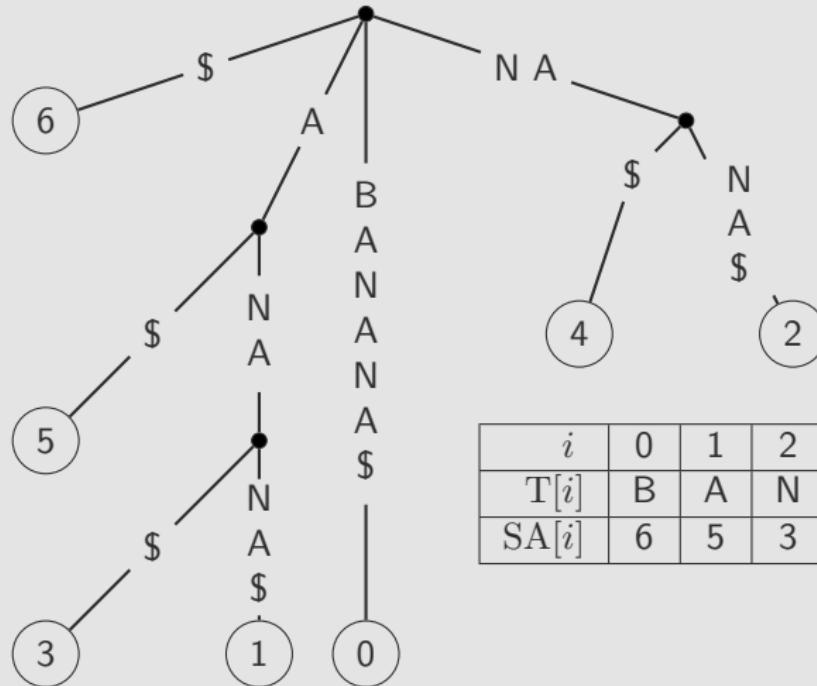
i	0	1	2	3	4	5	6
$\text{SA}[i]$	6	5	3	1	0	4	2
$\text{LCP}[i]$	-1	0	1	3	0	0	2
	\$	A	A	A	B	N	N
	\$	N	N	A	A	A	
	A	A	N		\$	N	
	\$	N	A			A	
		A	N				\$
	\$	A					
		\$					

- ▶ Array containing the size of the longest common prefix of $S_{\text{SA}[i-1]}$ and $S_{\text{SA}[i]}$ at position i .

Suffix Trees

Data Structures

A Compressed Trie

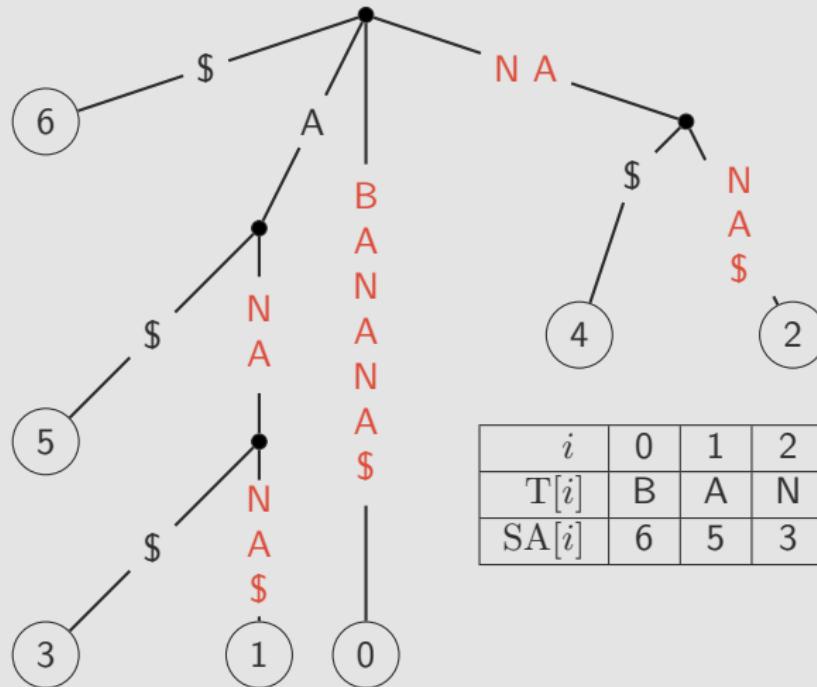


i	0	1	2	3	4	5	6
$T[i]$	B	A	N	A	N	A	\$
$SA[i]$	6	5	3	1	0	4	2

Suffix Trees

Data Structures

A Compressed Trie

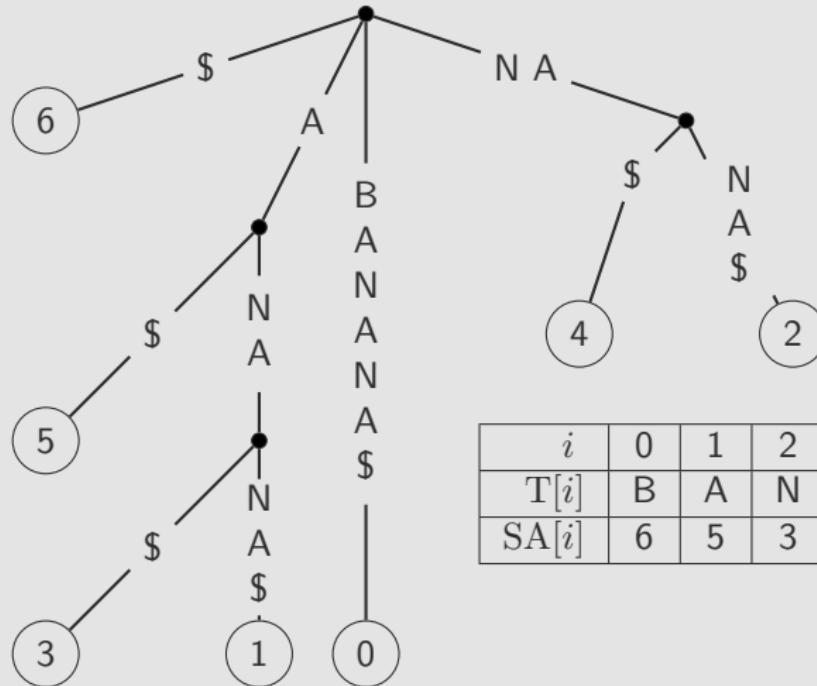


i	0	1	2	3	4	5	6
$T[i]$	B	A	N	A	N	A	\$
$SA[i]$	6	5	3	1	0	4	2

Suffix Trees

Data Structures

A Compressed Trie

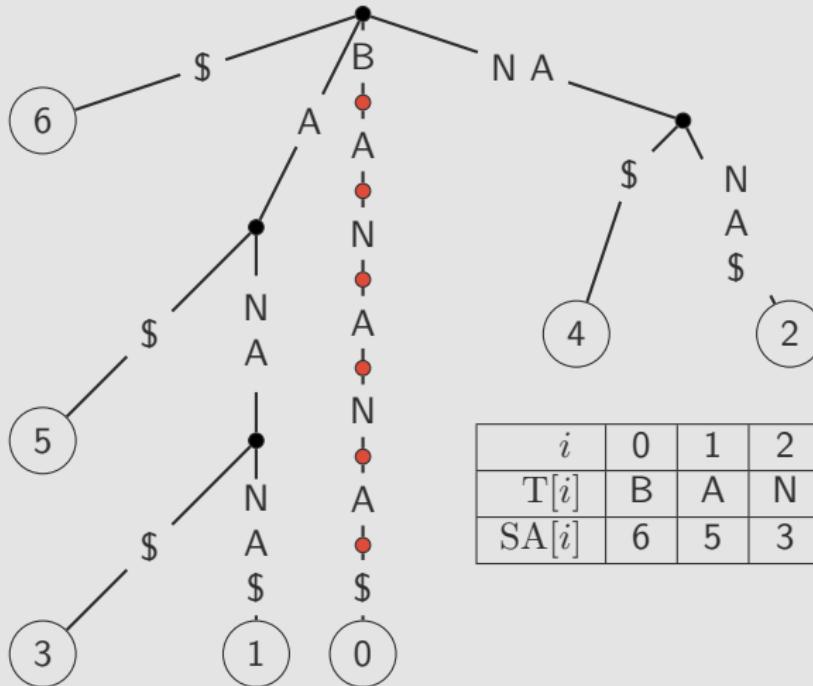


i	0	1	2	3	4	5	6
$T[i]$	B	A	N	A	N	A	\$
$SA[i]$	6	5	3	1	0	4	2

Suffix Trees

Data Structures

A Compressed Trie

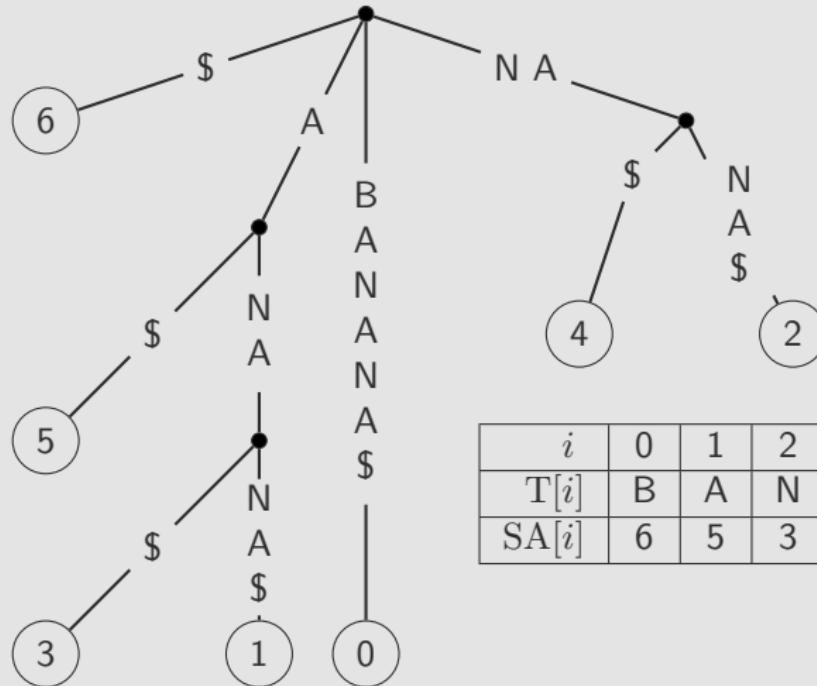


i	0	1	2	3	4	5	6
$T[i]$	B	A	N	A	N	A	\$
$SA[i]$	6	5	3	1	0	4	2

Suffix Trees

Data Structures

A Compressed Trie

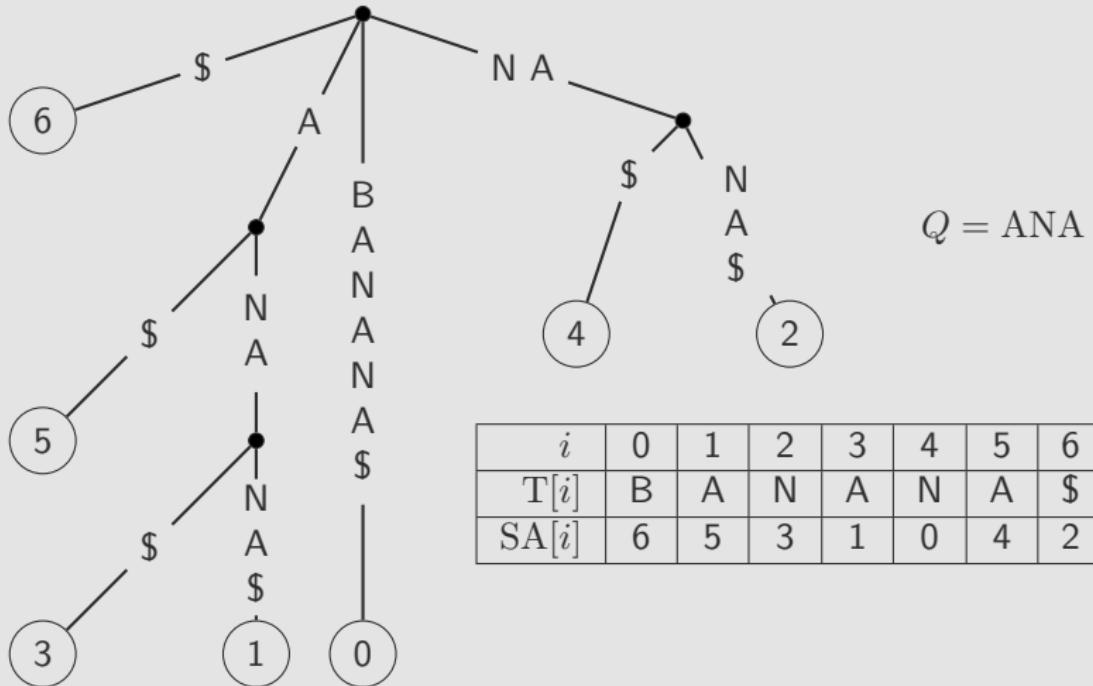


i	0	1	2	3	4	5	6
$T[i]$	B	A	N	A	N	A	\$
$SA[i]$	6	5	3	1	0	4	2

Suffix Trees

Data Structures

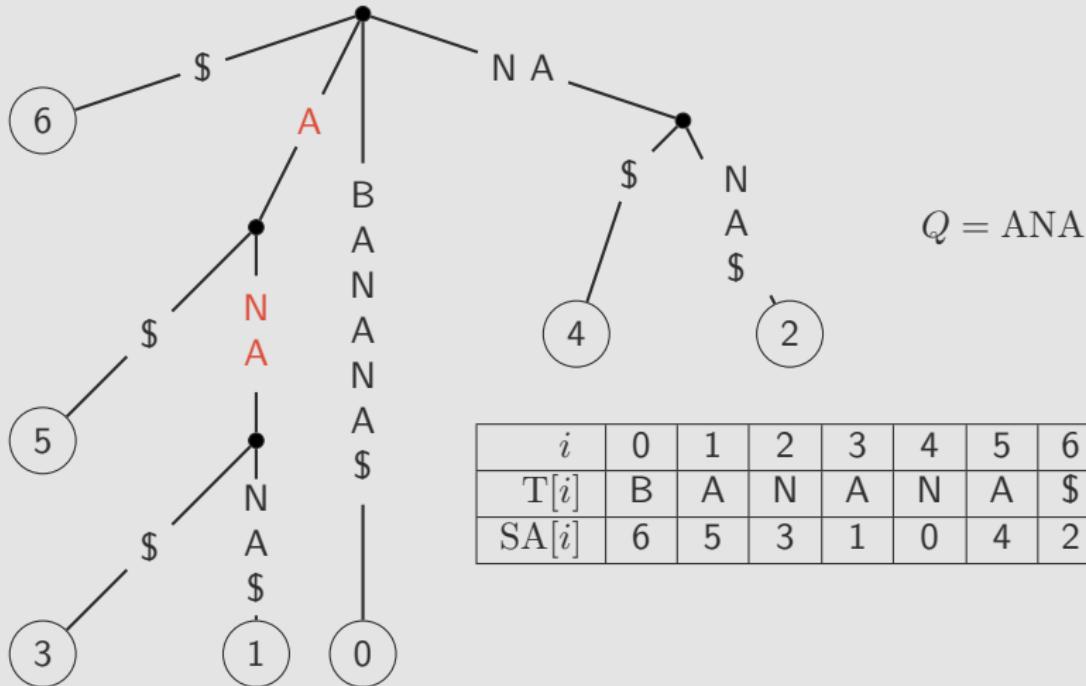
A Compressed Trie



Suffix Trees

Data Structures

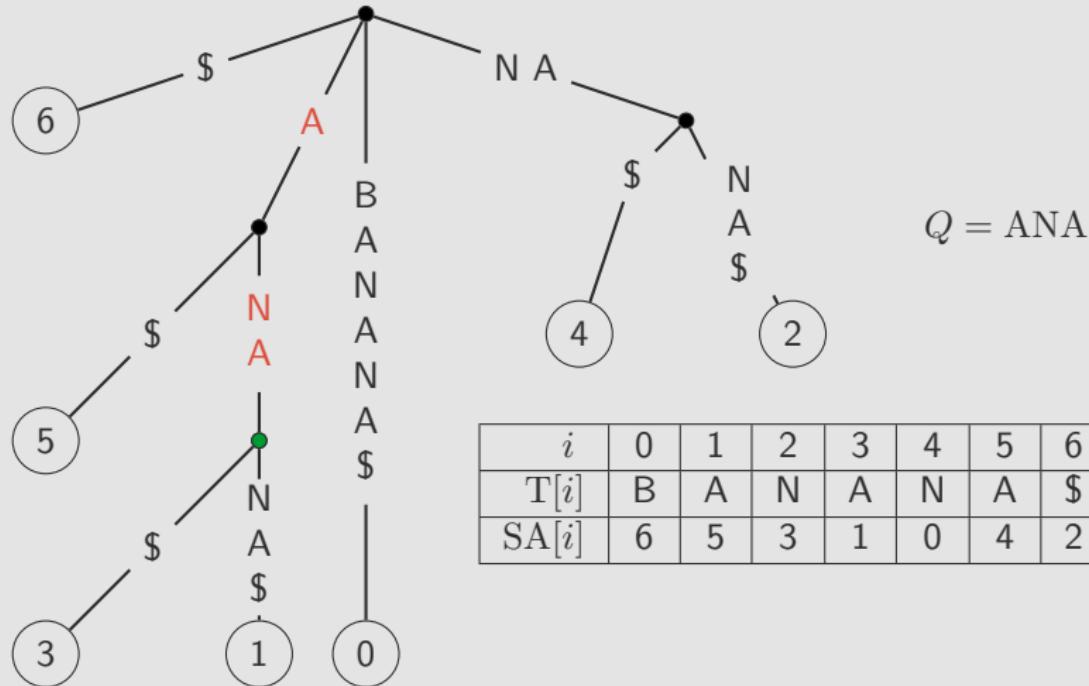
A Compressed Trie



Suffix Trees

Data Structures

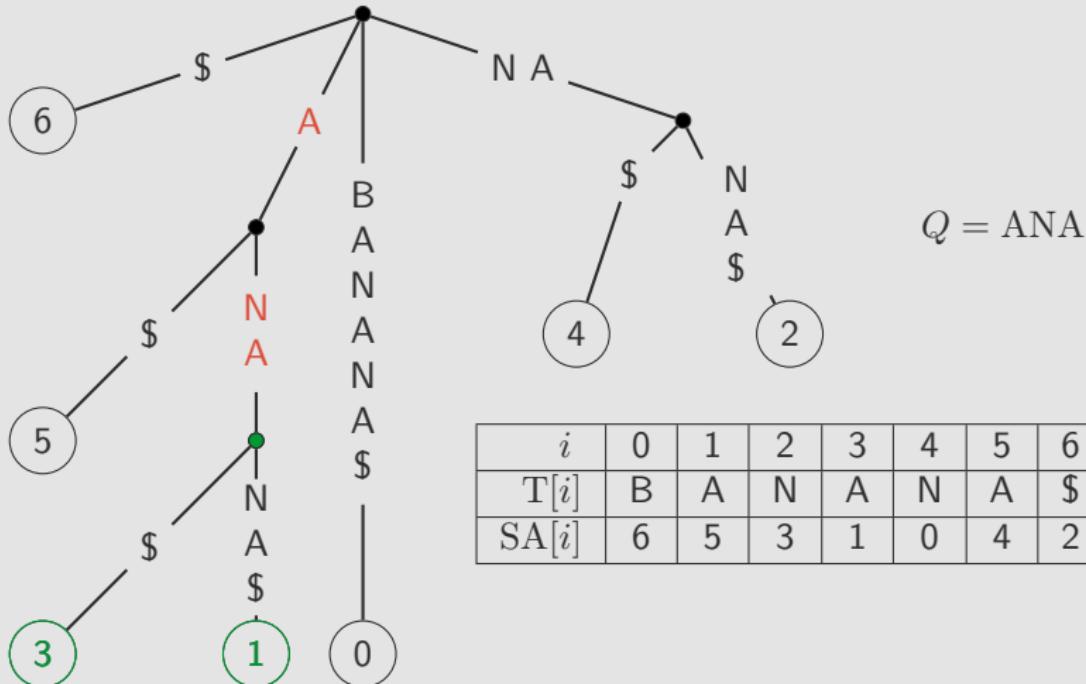
A Compressed Trie



Suffix Trees

Data Structures

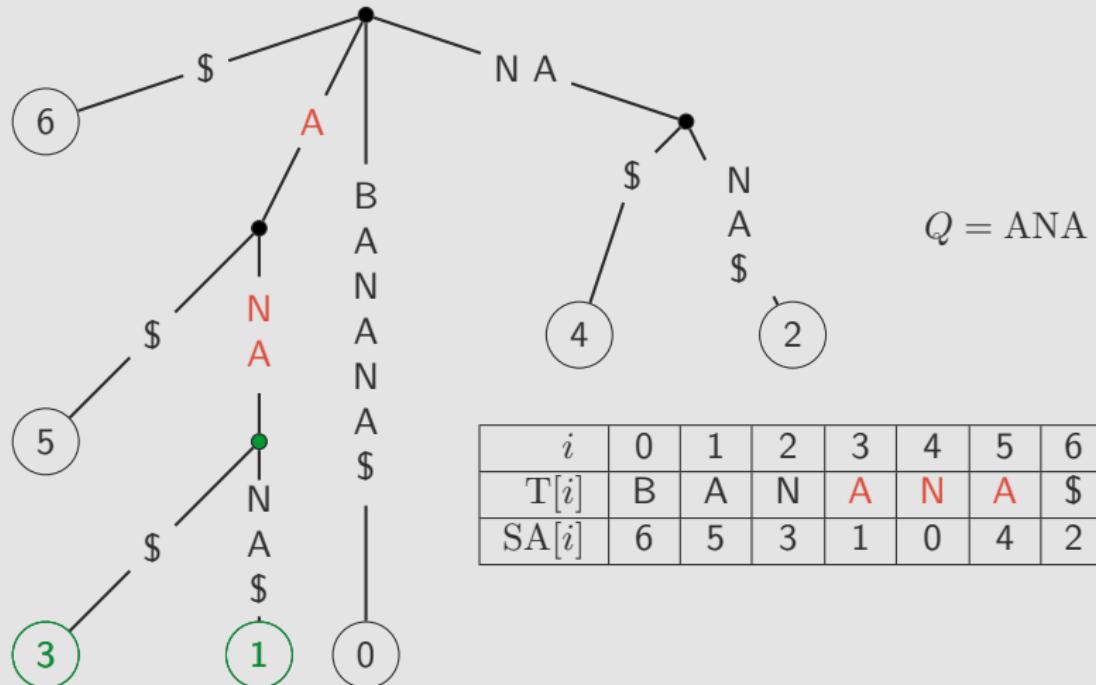
A Compressed Trie



Suffix Trees

Data Structures

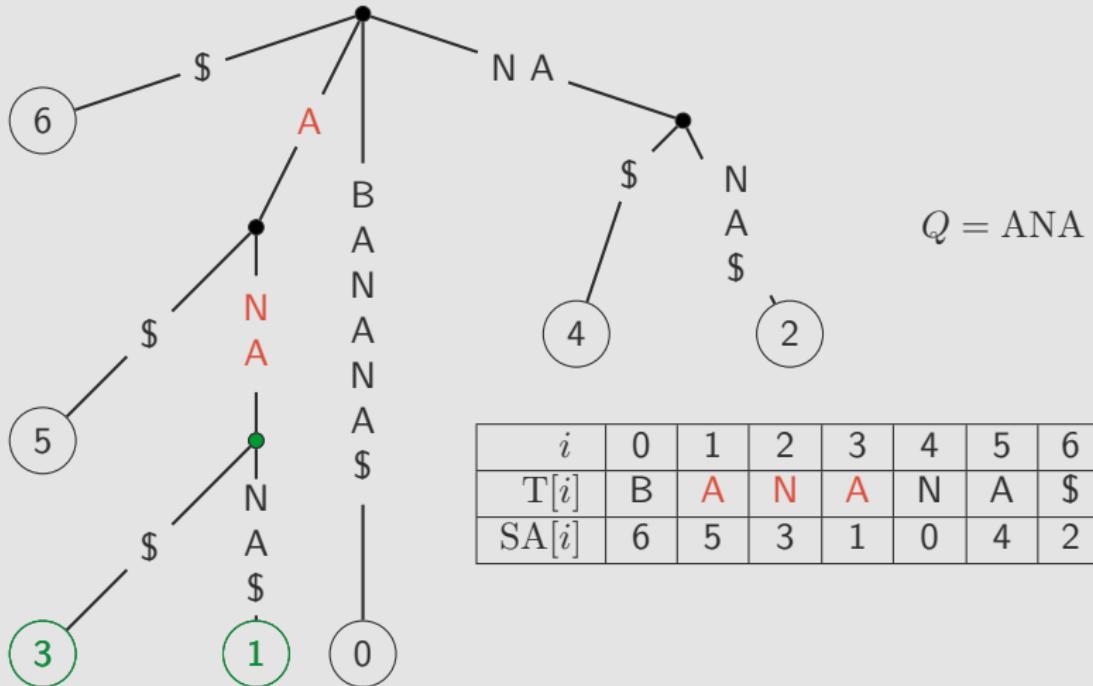
A Compressed Trie



Suffix Trees

Data Structures

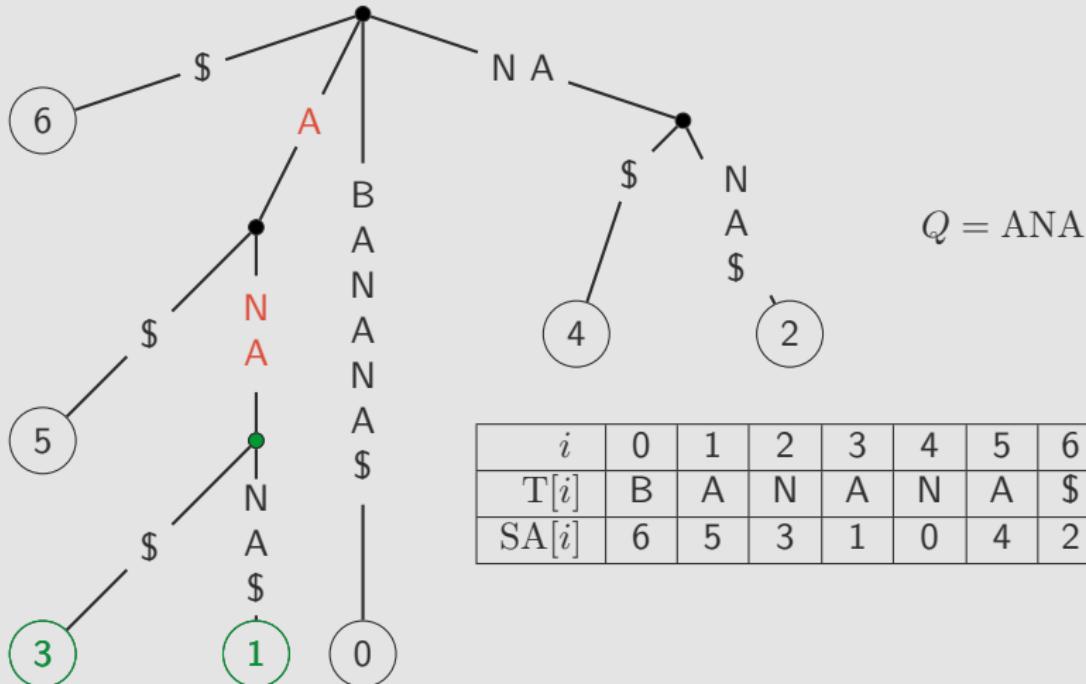
A Compressed Trie



Suffix Trees

Data Structures

A Compressed Trie



String B-Trees

The Extended Example



$T = \text{BAANNAANNAAA\$}$

B-Tree on top of a Suffixarray

SA[i] 11 10 9 5 1 6 2 0 8 4 7 3

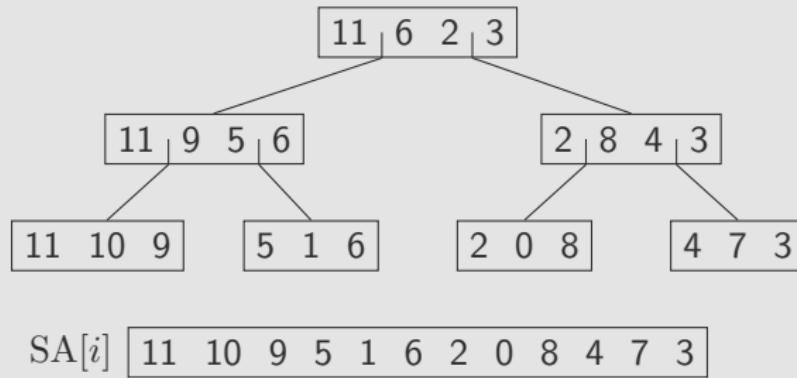
String B-Trees

The Extended Example



$T = \text{BAANNAANNAA\$}$

B-Tree on top of a Suffixarray



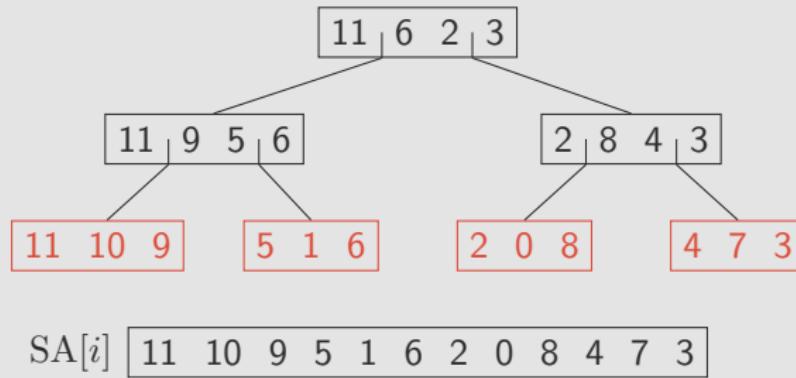
String B-Trees

The Extended Example



$T = \text{BAANNAANNAA\$}$

B-Tree on top of a Suffixarray



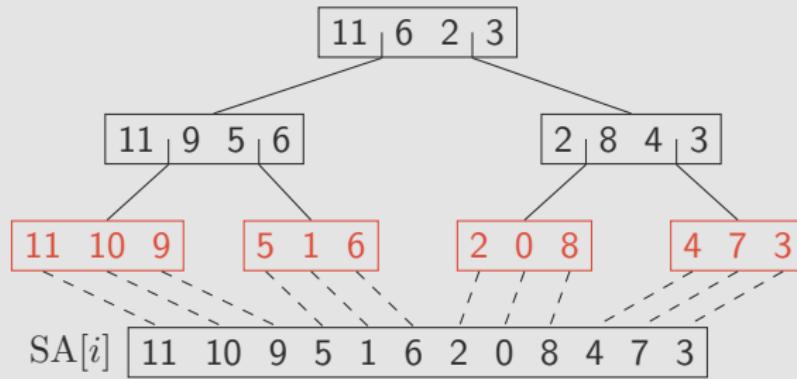
String B-Trees

The Extended Example



$T = \text{BAANNAANNAA\$}$

B-Tree on top of a Suffixarray



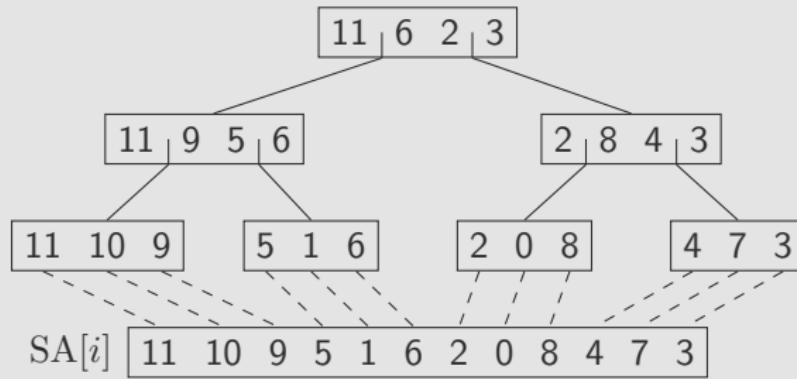
String B-Trees

The Extended Example



$T = \text{BAANNAANNAA\$}$

B-Tree on top of a Suffixarray



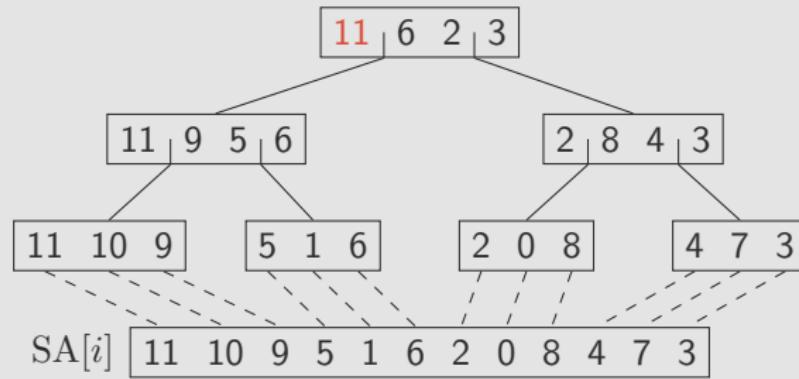
String B-Trees

The Extended Example



$T = \text{BAANNAANNAAA\$}$

B-Tree on top of a Suffixarray



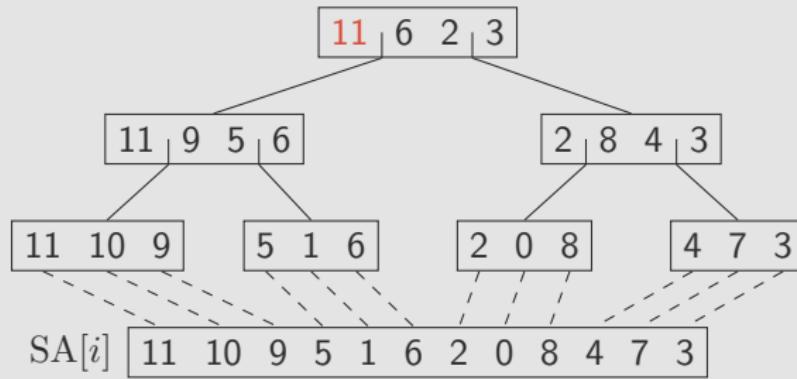
String B-Trees

The Extended Example



$T = \text{BAANNAANNAA\$}$

B-Tree on top of a Suffixarray



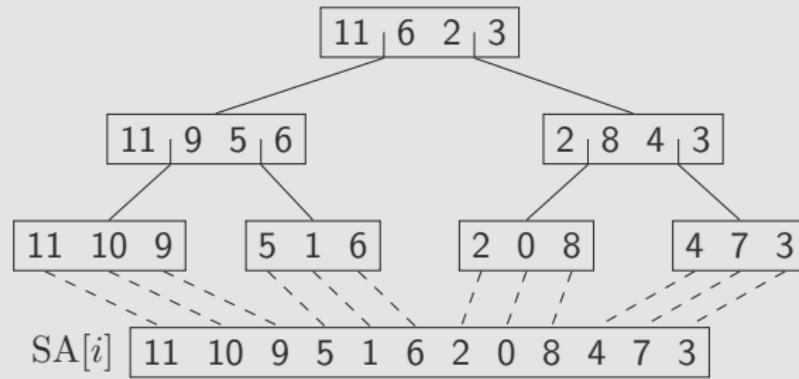
String B-Trees

The Extended Example



$T = \text{BAANNAANNAAA\$}$

B-Tree on top of a Suffixarray



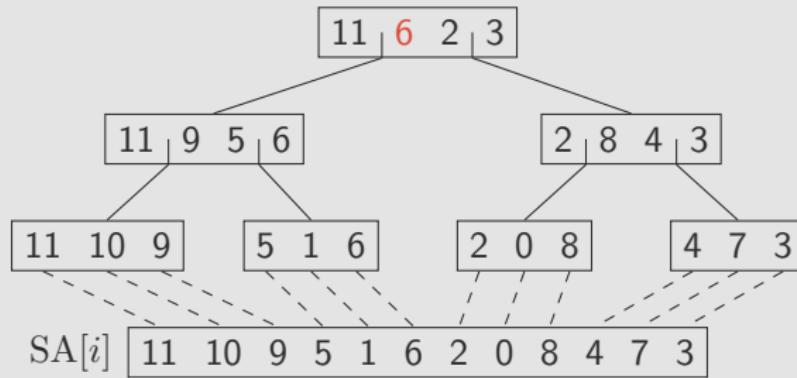
String B-Trees

The Extended Example



$T = \text{BAANNAANNAA\$}$

B-Tree on top of a Suffixarray



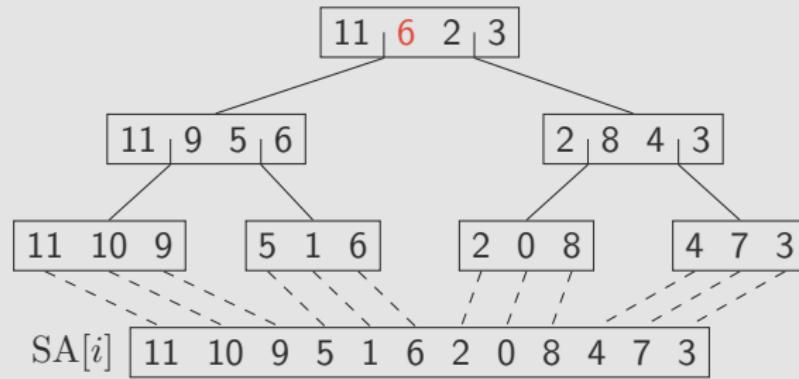
String B-Trees

The Extended Example



$T = \text{BAANNA} \underset{\text{red}}{\text{ANNAA}} \text{ \$}$

B-Tree on top of a Suffixarray



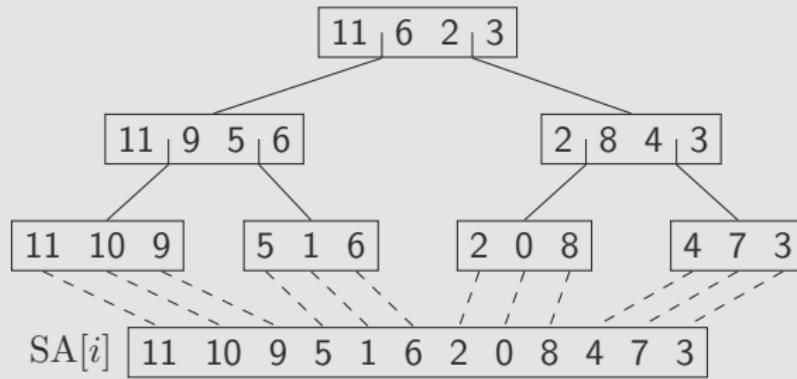
String B-Trees

The Extended Example



$T = \text{BAANNAANNAA\$}$

B-Tree on top of a Suffixarray



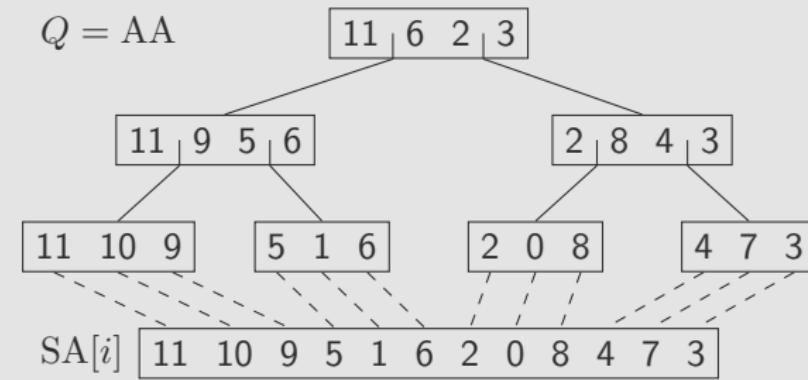
String B-Trees

The Extended Example



$T = \text{BAANNAANNAA\$}$

B-Tree on top of a Suffixarray



String B-Trees

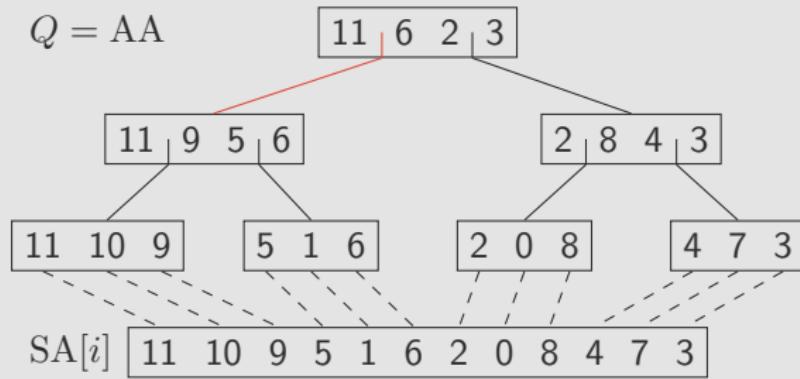
The Extended Example



$T = \text{BAANNAANNAA\$}$

B-Tree on top of a Suffixarray

$Q = \text{AA}$



String B-Trees

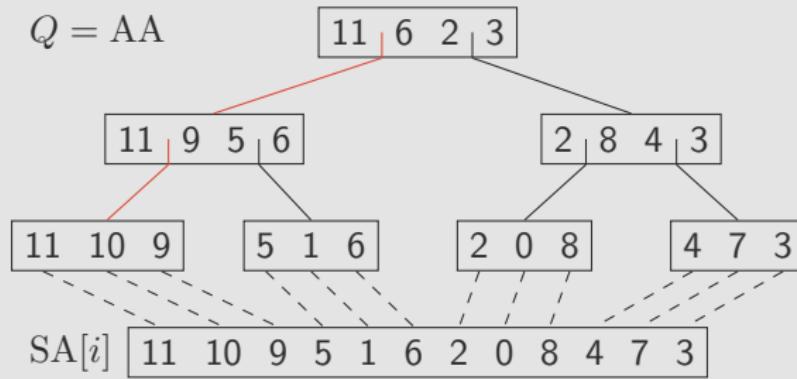
The Extended Example



$T = \text{BAANNAANNAAA\$}$

B-Tree on top of a Suffixarray

$Q = \text{AA}$



String B-Trees

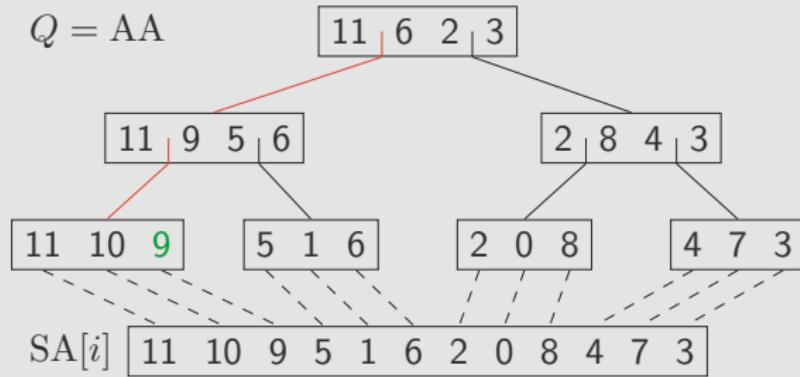
The Extended Example



$T = BAANNAANNAA \$$

B-Tree on top of a Suffixarray

$Q = AA$



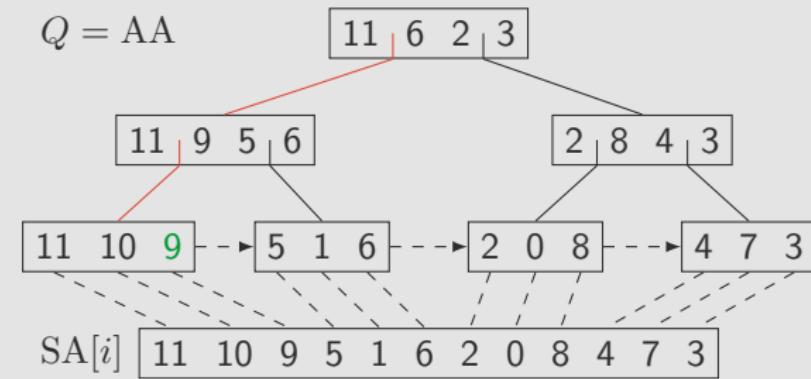
String B-Trees

The Extended Example



$T = \text{BAANNAANNAAA\$}$

B-Tree on top of a Suffixarray



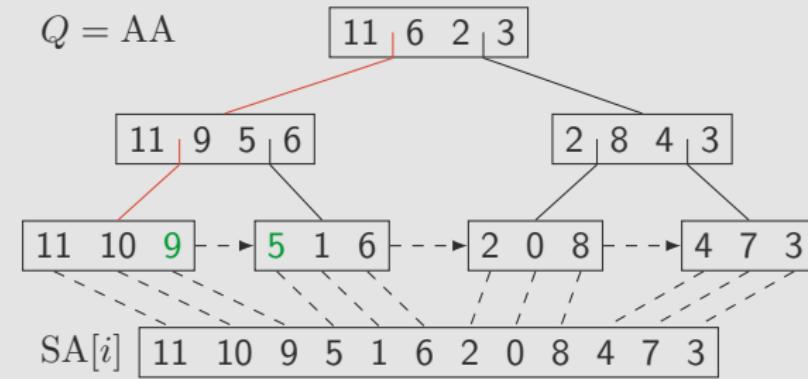
String B-Trees

The Extended Example



$T = \text{BAANNAANNAAA\$}$

B-Tree on top of a Suffixarray



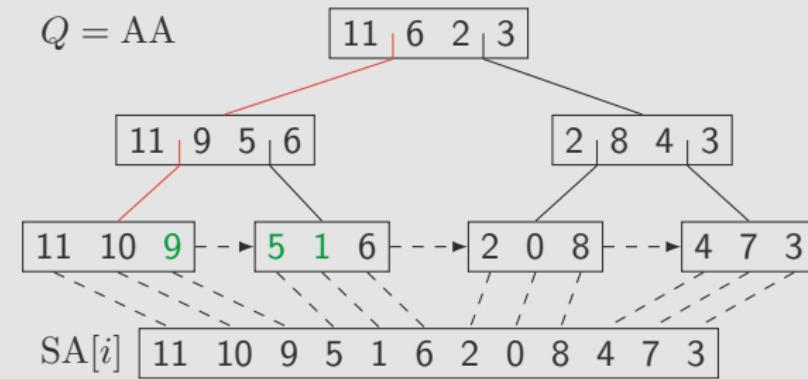
String B-Trees

The Extended Example



$T = \text{BAANNAANNAAA\$}$

B-Tree on top of a Suffixarray



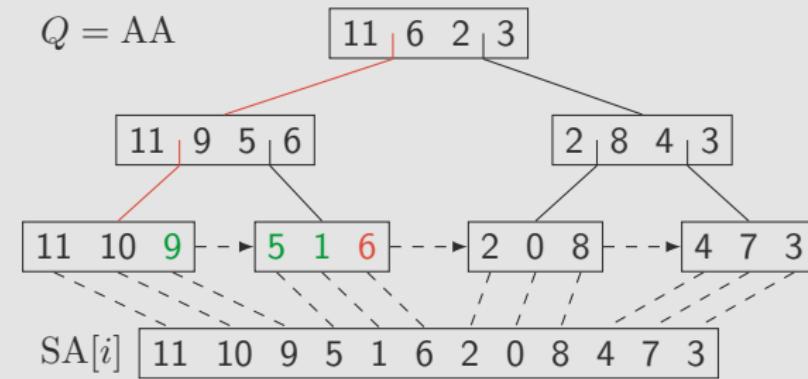
String B-Trees

The Extended Example



$T = \text{BAANNAANNAAA\$}$

B-Tree on top of a Suffixarray



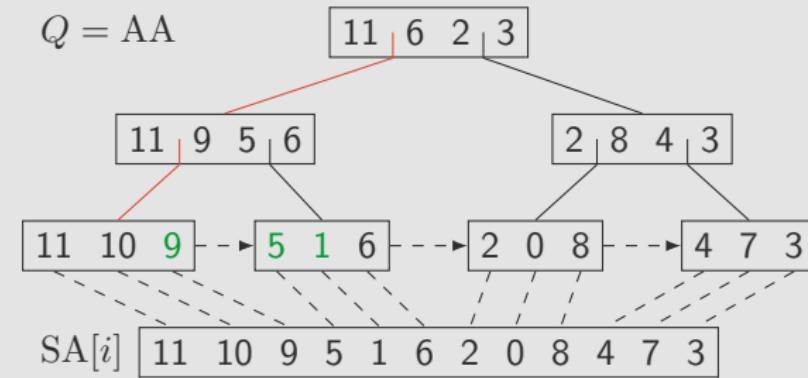
String B-Trees

The Extended Example



$T = \text{BAANNAAANNAA\$}$

B-Tree on top of a Suffixarray



Succinct Data Structures

The Idea

- ▶ Using minimum space close to the information-theoretic lower bound.
- ▶ Handle large problem sizes in the internal memory.

Succinct Data Structures

The Idea

- ▶ Using minimum space close to the information-theoretic lower bound.
- ▶ Handle large problem sizes in the internal memory.

Constant Time Operations on Bit Vectors

$\text{rank}_q(i) = \#q's \text{ up to position } i$

$\text{select}_q(i) = \text{position of } i^{\text{th}} \text{ } q$

0 1 1 1 0 0 1 0 1 0 0 1

Succinct Data Structures

The Idea

- ▶ Using minimum space close to the information-theoretic lower bound.
- ▶ Handle large problem sizes in the internal memory.

Constant Time Operations on Bit Vectors

$\text{rank}_q(i) = \#q's \text{ up to position } i \rightarrow \text{rank}_1(4) = 3$

$\text{select}_q(i) = \text{position of } i^{\text{th}} \text{ } q$

0 1 1 1 0 0 1 0 1 0 0 1

Succinct Data Structures

The Idea

- ▶ Using minimum space close to the information-theoretic lower bound.
- ▶ Handle large problem sizes in the internal memory.

Constant Time Operations on Bit Vectors

$\text{rank}_q(i) = \#q's \text{ up to position } i \rightarrow \text{rank}_1(4) = 3$

$\text{select}_q(i) = \text{position of } i^{\text{th}} q$

0 1 1 1 0 0 1 0 1 0 0 1

Succinct Data Structures

The Idea

- ▶ Using minimum space close to the information-theoretic lower bound.
- ▶ Handle large problem sizes in the internal memory.

Constant Time Operations on Bit Vectors

$\text{rank}_q(i) = \#q's \text{ up to position } i \rightarrow \text{rank}_1(4) = 3$

$\text{select}_q(i) = \text{position of } i^{\text{th}} q$

0	1	1	1	0	0	1	0	1	0	0	1
1	2	3									

Succinct Data Structures

The Idea

- ▶ Using minimum space close to the information-theoretic lower bound.
- ▶ Handle large problem sizes in the internal memory.

Constant Time Operations on Bit Vectors

$\text{rank}_q(i) = \#q's \text{ up to position } i \rightarrow \text{rank}_1(4) = 3$

$\text{select}_q(i) = \text{position of } i^{\text{th}} \text{ } q$

0 1 1 1 0 0 1 0 1 0 0 1

Succinct Data Structures

The Idea

- ▶ Using minimum space close to the information-theoretic lower bound.
- ▶ Handle large problem sizes in the internal memory.

Constant Time Operations on Bit Vectors

$\text{rank}_q(i) = \#q's \text{ up to position } i \rightarrow \text{rank}_1(4) = 3$

$\text{select}_q(i) = \text{position of } i^{\text{th}} q \rightarrow \text{select}_0(3) = 5$

0 1 1 1 0 0 1 0 1 0 0 1

Succinct Data Structures

The Idea

- ▶ Using minimum space close to the information-theoretic lower bound.
- ▶ Handle large problem sizes in the internal memory.

Constant Time Operations on Bit Vectors

$\text{rank}_q(i) = \#q's \text{ up to position } i \rightarrow \text{rank}_1(4) = 3$

$\text{select}_q(i) = \text{position of } i^{\text{th}} q \rightarrow \text{select}_0(3) = 5$

0 1 1 1 0 0 1 0 1 0 0 1

Succinct Data Structures

The Idea

- ▶ Using minimum space close to the information-theoretic lower bound.
- ▶ Handle large problem sizes in the internal memory.

Constant Time Operations on Bit Vectors

$\text{rank}_q(i) = \#q's \text{ up to position } i \rightarrow \text{rank}_1(4) = 3$

$\text{select}_q(i) = \text{position of } i^{\text{th}} q \rightarrow \text{select}_0(3) = 5$

0	1	1	1	0	0	1	0	1	0	0	1
0	1	2	3	4	5						

Succinct Data Structures

The Idea

- ▶ Using minimum space close to the information-theoretic lower bound.
- ▶ Handle large problem sizes in the internal memory.

Constant Time Operations on Bit Vectors

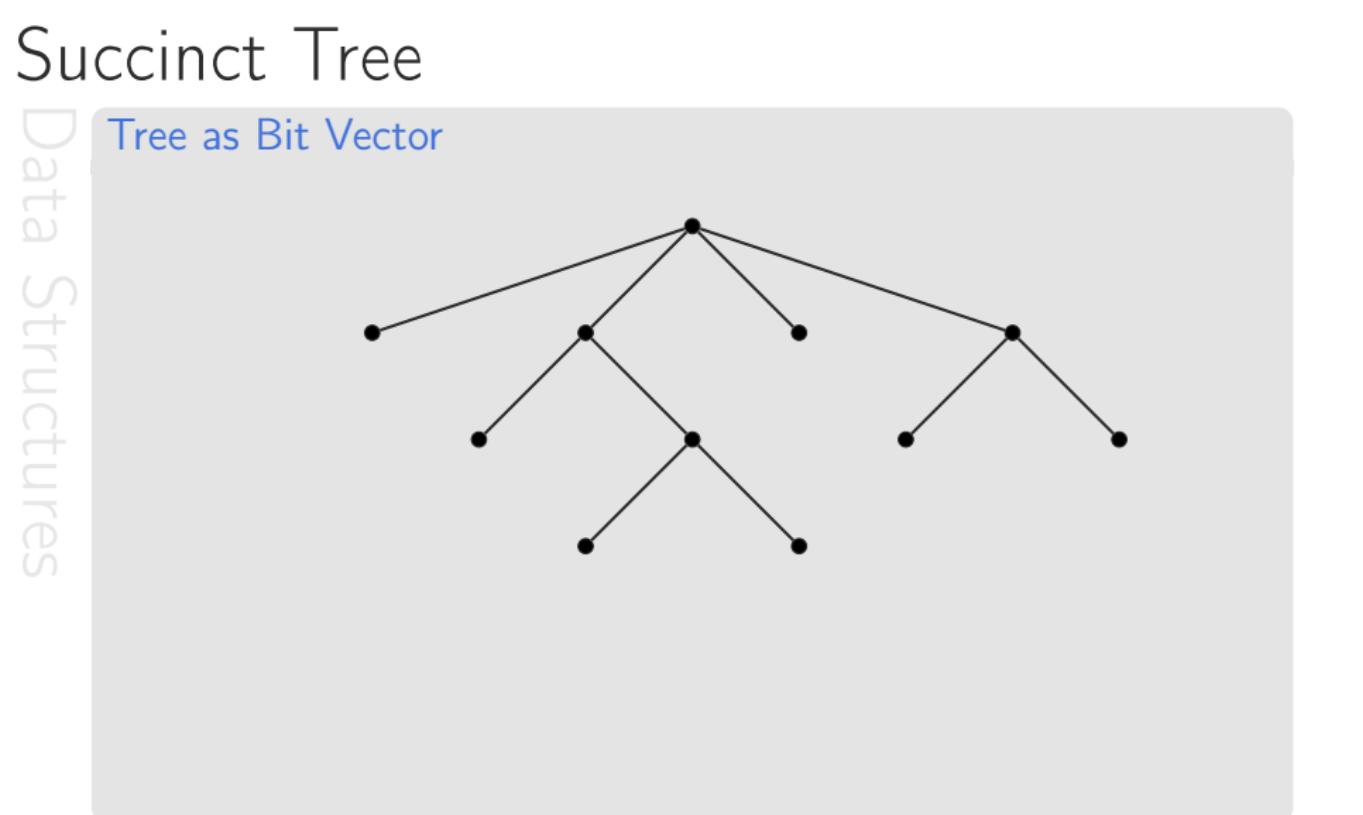
$\text{rank}_q(i) = \#q's \text{ up to position } i \rightarrow \text{rank}_1(4) = 3$

$\text{select}_q(i) = \text{position of } i^{\text{th}} q \rightarrow \text{select}_0(3) = 5$

0 1 1 1 0 0 1 0 1 0 0 1

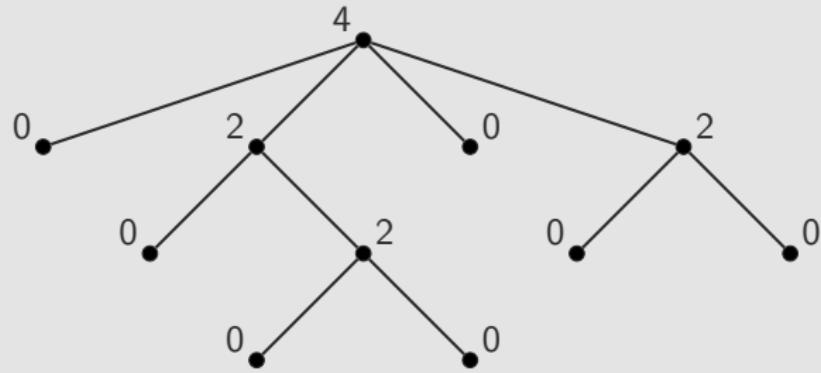
Example: Tree

- ▶ A tree $G = (V, E)$ can be represented as bit vector of length $2|V| + 1$.
- ▶ rank and select are available.



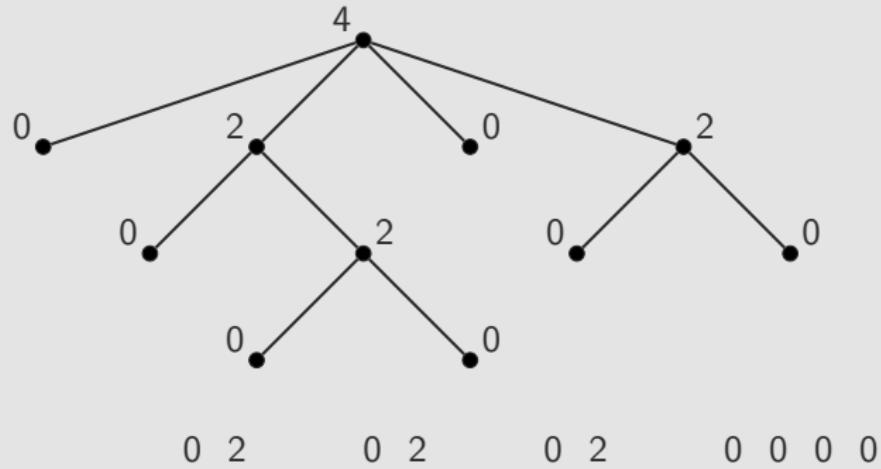
Succinct Tree

Tree as Bit Vector



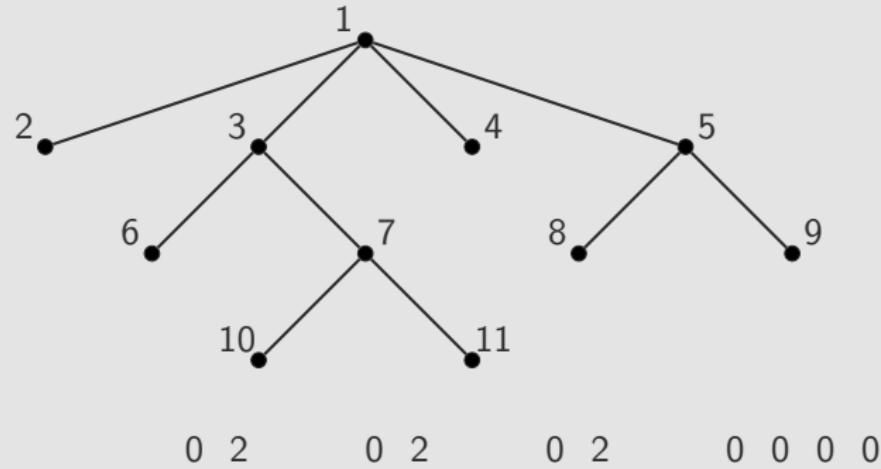
Succinct Tree

Tree as Bit Vector



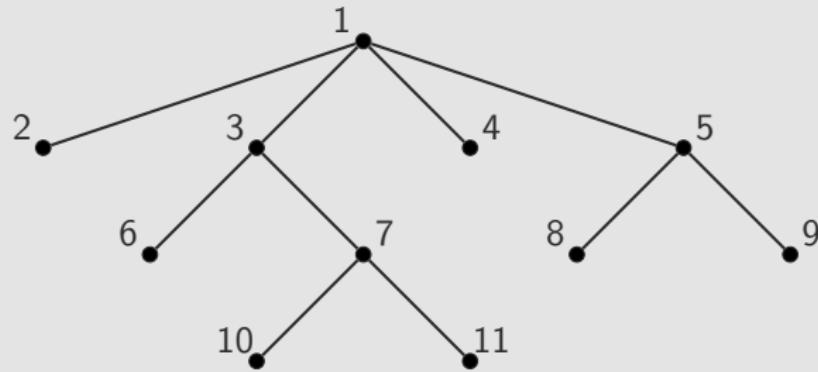
Succinct Tree

Tree as Bit Vector



Succinct Tree

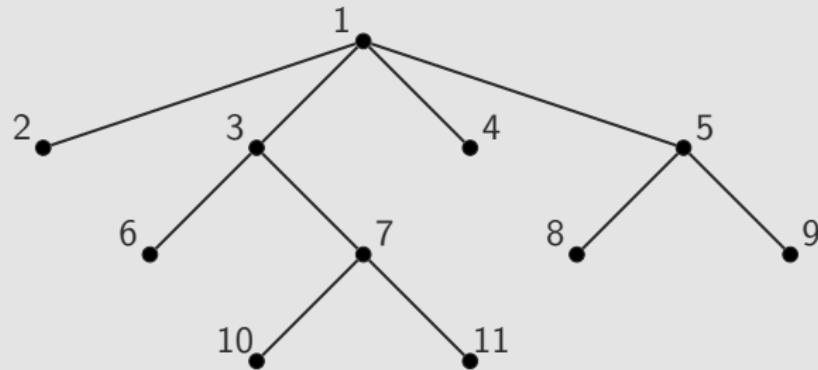
Tree as Bit Vector



4	0	2	0	2	0	2	0	0	0	0	0
1	1	1	0	0	1	1	0	0	1	1	0

Succinct Tree

Tree as Bit Vector

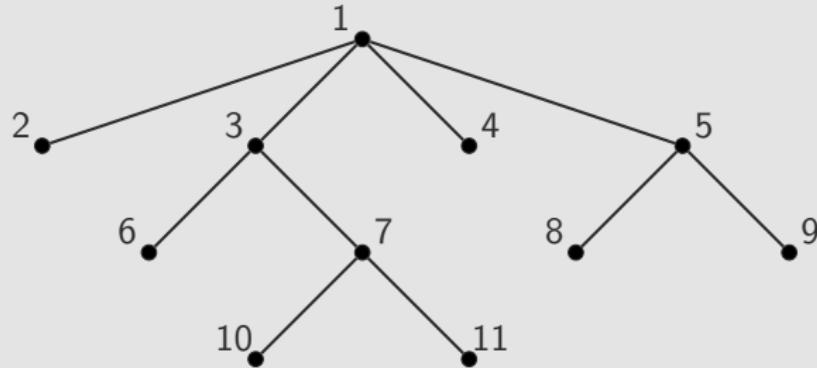


4	0	2	0	2	0	2	0	0	0	0	0
1	1	1	1	0	0	1	1	0	0	1	1
1	0	1	1	1	0	0	1	1	0	0	0

Succinct Tree

Data Structures

Tree as Bit Vector

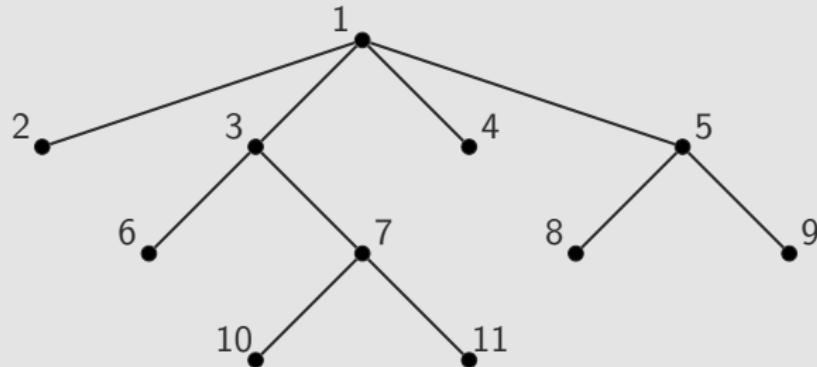


4	0	2	0	2	0	2	0	0	0	0	0
1	1	1	1	0	0	1	1	0	0	1	0
1	0	1	1	1	0	0	1	1	0	0	0

$$\text{parent}(v) = \text{rank}_0(\text{select}_1(v))$$

Succinct Tree

Tree as Bit Vector



4	0	2	0	2	0	2	0	2	0	0	0	0
1	1	1	1	0	0	1	1	0	0	1	1	0
1	0	1	1	1	1	0	0	1	1	0	0	0
1	1	2	3	4	5	2	3	6	7	4	5	8

10 11 18 9 10 11 12

$$\text{parent}(v) = \text{rank}_0(\text{select}_1(v))$$

State of the Art (1/2)

Internal Memory (RAM)

- ▶ SA, LCP and String B-trees are well studied.
- ▶ Construction in optimal time in theory.
- ▶ Well-performing practical implementations.

State of the Art (1/2)

Internal Memory (RAM)

- ▶ SA, LCP and String B-trees are well studied.
- ▶ Construction in optimal time in theory.
- ▶ Well-performing practical implementations.

External Memory

- ▶ Theoretical I/O-Optimal Algorithm exists for the SA.
- ▶ Recent work; still an active topic.
- ▶ LCP is needed for the construction of String B-Trees.

State of the Art (2/2)

Succinct Data Structures

- ▶ Working with large problem sizes.
- ▶ Almost exclusively only in internal memory.
- ▶ Practical implementation exists.
- ▶ Fast and space efficient constructions remain a major issue.

State of the Art (2/2)

Succinct Data Structures

- ▶ Working with large problem sizes.
- ▶ Almost exclusively only in internal memory.
- ▶ Practical implementation exists.
- ▶ Fast and space efficient constructions remain a major issue.

Parallel and Distributed Computing

- ▶ Parallel SA Algorithms (no implementations) at the end of the 80s.
- ▶ First practical implementations end of the end of last decade.
- ▶ GPGPU is possible and yields to better results.
- ▶ Large constant factor gap to the practical performance of the best sequential algorithms

Preliminary Work – External Memory

eSAIS [Bingmann et al., 2013]

- ▶ Based on SAIS: Linear time and fast in practice [Nong et al., 2009]
- ▶ Induced Sorting Principle.

Preliminary Work – External Memory

eSAIS [Bingmann et al., 2013]

- ▶ Based on SAIS: Linear time and fast in practice [Nong et al., 2009]
- ▶ Induced Sorting Principle.

Induced Sorting

i	0	1	2	3	4	5	6	7	8	9	10	11
$T[i]$	B	A	A	N	N	A	A	N	N	A	A	\$
c -Bucket	\$			A			B			N		

Preliminary Work – External Memory

eSAIS [Bingmann et al., 2013]

- ▶ Based on SAIS: Linear time and fast in practice [Nong et al., 2009]
- ▶ Induced Sorting Principle.

Induced Sorting

i	0	1	2	3	4	5	6	7	8	9	10	11
$T[i]$	B	A	A	N	N	A	A	N	N	A	A	\$
c -Bucket	\$			A			B			N		

Preliminary Work – External Memory

eSAIS [Bingmann et al., 2013]

- ▶ Based on SAIS: Linear time and fast in practice [Nong et al., 2009]
- ▶ Induced Sorting Principle.

Induced Sorting

i	0	1	2	3	4	5	6	7	8	9	10	11
$T[i]$	B	A	A	N	N	A	A	N	N	A	A	\$
c -Bucket	\$			A			B			N		

Preliminary Work – External Memory

eSAIS [Bingmann et al., 2013]

- ▶ Based on SAIS: Linear time and fast in practice [Nong et al., 2009]
- ▶ Induced Sorting Principle.
- ▶ Computes SA and LCP in external memory.

Induced Sorting

i	0	1	2	3	4	5	6	7	8	9	10	11
$T[i]$	B	A	A	N	N	A	A	N	N	A	A	\$
c -Bucket	\$			A			B			N		

Preliminary Work – External Memory

eSAIS [Bingmann et al., 2013]

- ▶ Based on SAIS: Linear time and fast in practice [Nong et al., 2009]
- ▶ Induced Sorting Principle.
- ▶ Computes SA and LCP in external memory.

Induced Sorting

i	0	1	2	3	4	5	6	7	8	9	10	11
$T[i]$	B	A	A	N	N	A	A	N	N	A	A	\$
c -Bucket	\$			A			B			N		

Results

- ▶ Outperforms all other external memory SA & LCP algorithms.
- ▶ 80 GigaByte XML dump with 4 Gigabyte main memory.
- ▶ 2.5 μ seconds per character and 18 TiB I/O-volume.

Preliminary Work – Currently In Progress

The Project

Parallelization of DivSufSort

- ▶ Based on DivSufsort: Linear time and fast in practice [Mori, 2005]
- ▶ Induced Sorting Principle.
- ▶ Semi-parallelization possible.
- ▶ Computes SA and LCP.

Preliminary Work – Currently In Progress

Parallelization of DivSufSort

- ▶ Based on DivSufsort: Linear time and fast in practice [Mori, 2005]
- ▶ Induced Sorting Principle.
- ▶ Semi-parallelization possible.
- ▶ Computes SA and LCP.

Other Topics

- ▶ Bulk-parallel priority queue for SA and LCP construction.
- ▶ Hierarchic (integer) sorting on a really big cluster.

Objectives

Text indexing algorithms are not fit for data sizes > low terabyte regions.

Objectives

Text indexing algorithms are not fit for data sizes > low terabyte regions.

Key Objectives

1. Full use of local computing power:

Objectives

Text indexing algorithms are not fit for data sizes > low terabyte regions.

Key Objectives

1. Full use of local computing power:
 - ▶ Multicore and/or GPGPU

Objectives

Text indexing algorithms are not fit for data sizes > low terabyte regions.

Key Objectives

1. Full use of local computing power:
 - ▶ Multicore and/or GPGPU
 - ▶ Succinct Data Structures

Objectives

Text indexing algorithms are not fit for data sizes > low terabyte regions.

Key Objectives

1. Full use of local computing power:
 - ▶ Multicore and/or GPGPU
 - ▶ Succinct Data Structures
 - ▶ External Memory

Objectives

Text indexing algorithms are not fit for data sizes > low terabyte regions.

Key Objectives

1. Full use of local computing power:
 - ▶ Multicore and/or GPGPU
 - ▶ Succinct Data Structures
 - ▶ External Memory
2. Computing text indices in a distributed environment.

Objectives

Text indexing algorithms are not fit for data sizes > low terabyte regions.

Key Objectives

1. Full use of local computing power:
 - ▶ Multicore and/or GPGPU
 - ▶ Succinct Data Structures
 - ▶ External Memory
2. Computing text indices in a distributed environment.
3. Later: Higher query throughput of the resulting data structures.

Objectives

Text indexing algorithms are not fit for data sizes > low terabyte regions.

Key Objectives

1. Full use of local computing power:
 - ▶ Multicore and/or GPGPU
 - ▶ Succinct Data Structures
 - ▶ External Memory
2. Computing text indices in a distributed environment.
3. Later: Higher query throughput of the resulting data structures.

Thank You!